Tagged–photon events in polarized DIS process

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Abstract

Deep–inelastic events for the scattering of the longitudinally polarized electron by polarized proton with tagged collinear photon radiated from initial–state electron are considered. The corresponding cross–section is derived in the Born approximation. The model–independent radiative corrections to the Born cross–section are also calculated. Obtained result is applied to the case of elastic scattering.

1 Introduction

The idea to use radiative events in lepton–hadron interaction to expand the experimental possibilities for studies of different topics in the high–energy physics has become quite attractive in the last years.

Photon radiation from the initial e^+e^-–state, in the events with missing energy, has been successfully used at LEP for the measurement of the number of light neutrinos and for search the new physics signals [1]. The possibility to undertake the bottonium spectroscopy studies at B–factories by using emission of a hard photon from the electron or the positron has been considered in Ref. [2]. The important physical problem of the the total hadronic cross–section scanning in the electron–positron annihilation process at low and intermediate energies by means of the initial–state radiative events has been discussed widely in Ref. [3].

The initial–state collinear radiation is very important in certain regions of the deep–inelastic scattering (DIS) at $HERA$ kinematic domain. It leads to reduction of the projectile electron energy and therefore to a shift of the effective Bjorken variables in the hard scattering process as compared to those determined from the actual measurement of the scattered electron alone. That is why the radiative events in the DIS process

$$e^-(k_1) + p(p_1) \rightarrow e^-(k_2) + \gamma(k) + X(p_x) \tag{1}$$

have to be carefully taken into account [4].

Besides, the measurement of the energy of the photon emitted very close to the incident electron beam direction [5, 6] permits to overlap the kinematical region of photoproduction ($Q^2 = -(k_1 - k_2)^2 \simeq 0$) and DIS region with small trasferred momenta (Q^2 about a few GeV^2) within the high–energy $HERA$ experiments. These radiative events may be used also to determine independently the proton structure functions F_1 and F_2 in a single run without lowering the beam energy [5, 7]. The high–precision calculation (taking into account radiative corrections (RC)) of the corresponding cross–section has been performed in Ref. [8].

In the present paper we investigate the deep–inelastic events for the radiative process (1) with longitudinally polarized electron beam and polarized proton as a target. We suggest that, as in Ref. [8], the hard photon is emitted very close to the direction of the incoming electron beam ($\theta_\gamma = \widehat{\mathbf{p_1}\mathbf{k_1}} \leq \theta_0$, $\theta_0 \ll 1$), and the photon detector (PD) measures the energy of all photons inside the narrow cone with the opening angle 2θ around the electron beam. Simultaneously the scattered–electron 3–momentum is fixed.

We consider the cases of the longitudinal (along the electron beam direction) and perpendicular (in the plane $(\mathbf{k_1}, \mathbf{k_2})$) polarizations of the proton. In Section 2 we derive the corresponding cross–sections in the Born approximation and in Section 3 we calculate the different contributions into RC to the Born cross–section. The total radiative correction for different (exclusive and calorimeter) experimental conditions for the scattered–electron measurement is given in Section 4. Our results can be applied to describe the cross–section of the process (1) with target proton at rest as well as with colliding electron–proton beams. In Section 5 we apply the obtained in Section 4 results to describe the quasi–elastic scattering using the connection between the spin–dependent proton structure functions and the proton electromagnetic form factors in this limiting case.

2 Born approximation

The spin–independent part of the DIS cross–section with considered here experimental set–up has been investigated recently in details [8]. Now we consider the spin–dependent part of the corresponding cross–section that is described by means of the proton structure functions g_1 and g_2. As the opening angle of the forward PD is very small, and we consider only the cross–section where the tagged photon is integrated over the solid angle covered by PD, we can apply the quasi–real electron method [9] and parametrize these radiative events using the standard Bjorken variables

$$x = \frac{Q^2}{2p_1(k_1 - k_2)} \ , \quad y = \frac{2p_1(k_1 - k_2)}{V} \ , \quad V = 2p_1 k_1 \ , \tag{2}$$

and the energy fraction of the electron after the initial–state radiation of a collinear photon

$$z = \frac{2p_1(k_1 - k)}{V} = \frac{\varepsilon - \omega}{\varepsilon} \ , \tag{3}$$

where ε is the initial–electron energy and ω is the energy deposited in PD.

An alternative set of the kinematic variables, that is specially adapted to the case of the collinear–photon radiation, is given by the shifted Bjorken variables [5, 10]

$$\widehat{Q}^2 = -(k_1 - k_2 - k)^2 \ , \quad \hat{x} = \frac{\widehat{Q}^2}{2p_1(k_1 - k_2 - k)} \ , \quad \hat{y} = \frac{2p_1(k_1 - k_2 - k)}{2p_1(k_1 - k)} \ . \tag{4}$$

The relation between the shifted and standard Bjorken variables reads

$$\widehat{Q}^2 = zQ^2 \ , \quad \hat{x} = \frac{xyz}{z + y - 1} \ , \quad \hat{y} = \frac{z + y - 1}{z} \ . \tag{5}$$

At fixed values of x and y the lower limit of z can be derived from constraint on the shifted variable $\hat{x}$

$$\hat{x} < 1 \quad \rightarrow \quad z > \frac{1-y}{1-xy} \ .$$

In the framework of the Born approximation we use the following determination of the DIS cross–section in the radiative process (1) in terms of the contraction of the leptonic and hadronic tensors (further we will interest with the spin–dependent part of the cross–section only)

$$\frac{d\sigma}{\hat{y}d\hat{x}d\hat{y}} = \frac{4\pi\alpha^2(\widehat{Q}^2)}{\widehat{Q}^4} z L^B_{\mu\nu} H_{\mu\nu} \ , \tag{6}$$

where $\alpha(\widehat{Q}^2)$ is the running electromagnetic coupling constant, that takes into account the effects of the vacuum polarization, and the Born leptonic current tensor in considered case reads [11]

$$L^B_{\mu\nu} = \frac{\alpha}{4\pi^2} \int_\Omega \frac{d^3k}{\omega} 2i\varepsilon_{\mu\nu\lambda\rho}q_\lambda(k_{1\rho}R_t + k_{2\rho}R_s) \ , \quad q = k_1 - k_2 - k \ , \tag{7}$$

where Ω covers the solid angle of PD.

For the case of the initial–state collinear radiation, which we cosider in this paper, quantities R_t and R_s can be written as follows

$$R_t = -\frac{1}{(1-z)t} - \frac{2m^2}{t^2} \ , \quad R_s = -\frac{z}{(1-z)t} + \frac{2m^2(1-z)}{t^2} \ , \quad t = -2kk_1 \ , \quad q = zk_1 - k_2 \ . \tag{8}$$

The trivial angular integration of the Born leptonic tensor gives in accordance with the quasi–real electron approximation [9]

$$L^B_{\mu\nu} = \frac{\alpha}{2\pi} P(z, L_0) dz i\varepsilon_{\mu\nu\lambda\rho}q_\lambda k_{1\rho} \ , \quad L_0 = \ln\frac{\varepsilon^2\theta_0^2}{m^2} \ , \tag{9}$$

$$P(z, L_0) = \frac{1+z^2}{1-z}L_0 - \frac{2(1-z+z^2)}{1-z} \ ,$$

where m is the electron mass.

We write the spin–dependent part of the hadronic tensor, on the right side of Eq. (6), in the following form

$$H_{\mu\nu} = -iM\frac{\varepsilon_{\mu\nu\lambda\rho}q_\lambda}{2p_1q}[(g_1 + g_2)S_\rho - g_2\frac{Sq}{p_1q}p_{1\rho}] \ , \tag{10}$$

where M is the proton mass and S is the 4–vector of the proton polarization. When writing the expressions (7) and (10), we suppose that the polarization degree of both the electron and the proton equals to 1.

Our normalization is such that the proton structure functions g_1 and g_2 are dimensionless and in the limiting case of the elastic scattering ($\hat{x} \rightarrow 1$) they are expressed in terms of the proton electric (G_E) and magnetic (G_M) form factors as follows

$$g_1(\hat{x}, \widehat{Q}^2) \rightarrow \delta(1-\hat{x})[G_M G_E + \frac{\lambda}{1+\lambda}(G_M - G_E)G_M] \ , \quad \lambda = \frac{\widehat{Q}^2}{4M^2} \ , \tag{11}$$

$$g_2(\hat{x}, \widehat{Q}^2) \rightarrow -\delta(1-\hat{x})\frac{\lambda}{1+\lambda}(G_M - G_E)G_M \ , \quad G_{M,E} = G_{M,E}(\widehat{Q}^2) \ .$$

In our problem it is convenient to parametrize the 4–vector of the proton polarization in terms of the 4–momenta of the reaction under study [12]

$$S^{\parallel}_{\mu} = \frac{2M^2 k_{1\mu} - V p_{1\mu}}{MV} , \quad S^{\perp}_{\mu} = \frac{u p_{1\mu} + V k_{2\mu} - [2u\tau + V(1-y)]k_{1\mu}}{\sqrt{-uV^2(1-y) - u^2 M^2}} , \quad (12)$$

where $u = -Q^2$, $\tau = M^2/V$ and we neglect the electron mass here. The 4–vector of the longitudinal proton polarization has components

$$S^{\parallel}_{\mu} = (0, \mathbf{n_1}) , \quad S^{\parallel}_{\mu} = \left(-\frac{|\mathbf{p_1}|}{M}, \frac{\mathbf{n_1} E_1}{M}\right) \quad (13)$$

for the target at rest and colliding beams, respectively. Here $E_1(\mathbf{p_1})$ is the proton energy (3–momentum) and $\mathbf{n_1}$ is the unit vector along the initial–electron 3–momentum direction. The 4–vector of the perpendicular proton polarization $S^{\perp}_{\mu}$ is the same for both these cases

$$S^{\perp}_{\mu} = \left(0, \frac{\mathbf{n_2} - \mathbf{n_1}(\mathbf{n_1 n_2})}{\sqrt{1 - (\mathbf{n_1 n_2})^2}}\right) , \quad (14)$$

where $\mathbf{n_2}$ is the unit vector along the scattered–electron 3–momentum direction. It is easy to verify that $S^{\parallel} S^{\perp} = 0$.
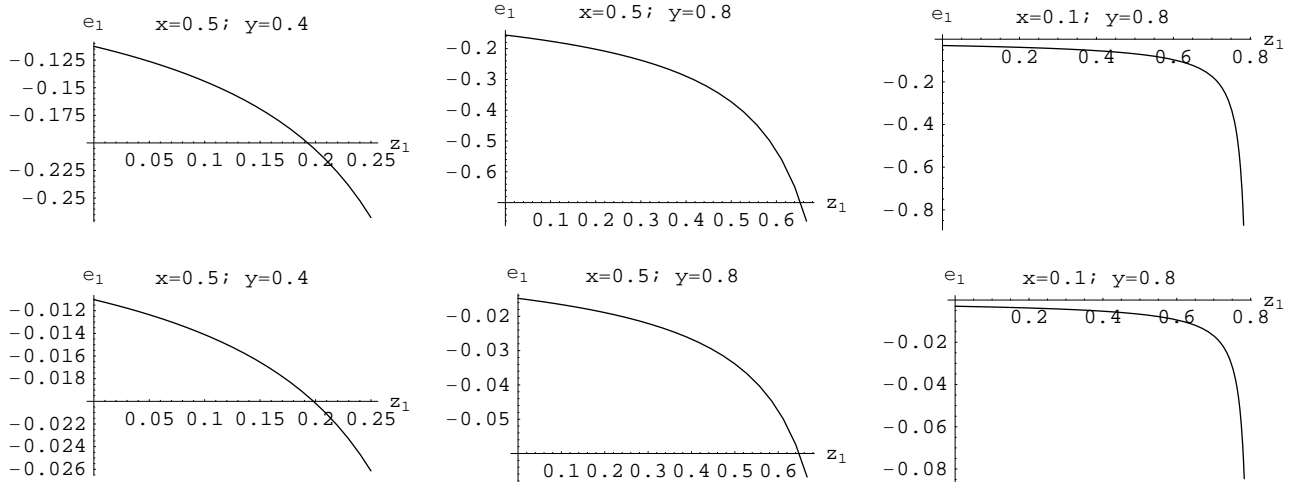

Figure 1: The dependence of the quantity e_1 on the energy fraction of the tagged photon $z_1 = 1 - z$ for different values of x, y and V. The upper set corresponds to $V = 10 \; GeV^2$ and the lower one to $V = 100 \; GeV^2$. The maximum value of z_1 is $y(1-x)/(1-xy)$.

Using the definitions of the DIS cross–section (6), leptonic and hadronic tensors (9), (10) and parametrization of the proton polarization (12), after simple calculations, we derive the spin–dependent part of the cross–section of the process (1), with tagged collinear photon radiated from initial state, in the following form

$$\frac{d\sigma^B_{\parallel,\perp}}{\hat{y} d\hat{x} d\hat{y} dz} = \frac{\alpha}{2\pi} P(z, L_0) \Sigma_{\parallel,\perp}(\hat{x}, \hat{y}, \widehat{Q}^2) , \quad (15)$$

4

$$\Sigma_\parallel = \frac{4\pi\alpha^2(\widehat{Q}^2)}{\widehat{V}\hat{y}}(\hat{\tau} - \frac{2-\hat{y}}{2\hat{x}\hat{y}})g_1(\hat{x},\widehat{Q}^2)[1 + e_1\hat{R}(\hat{x},\widehat{Q}^2)] \ , \qquad (16)$$

$$\Sigma_\perp = -\frac{4\pi\alpha^2(\widehat{Q}^2)}{\widehat{V}\hat{y}}\sqrt{\frac{M^2}{\widehat{Q}^2}(1-\hat{y}-\hat{x}\hat{y}\hat{\tau})}g_1(\hat{x},\widehat{Q}^2)[1 + e_2\hat{R}(\hat{x},\widehat{Q}^2)] \ , \qquad (17)$$

where

$$e_1 = \frac{4\hat{\tau}\hat{x}}{2\hat{x}\hat{y}\hat{\tau} + \hat{y} - 2} \ , \quad e_2 = \frac{2}{\hat{y}} \ , \quad \hat{R} = \frac{g_2(\hat{x},\widehat{Q}^2)}{g_1(\hat{x},\widehat{Q}^2)} \ , \quad \hat{\tau} = \frac{M^2}{\widehat{V}} \ , \quad \widehat{V} = zV \ .$$

It is useful to remind that unpolarized DIS cross–section is proportional to $\sigma_T(1 + eR)$, where $R = \sigma_L/\sigma_T$ and for events with the tagged collinear photon [5]
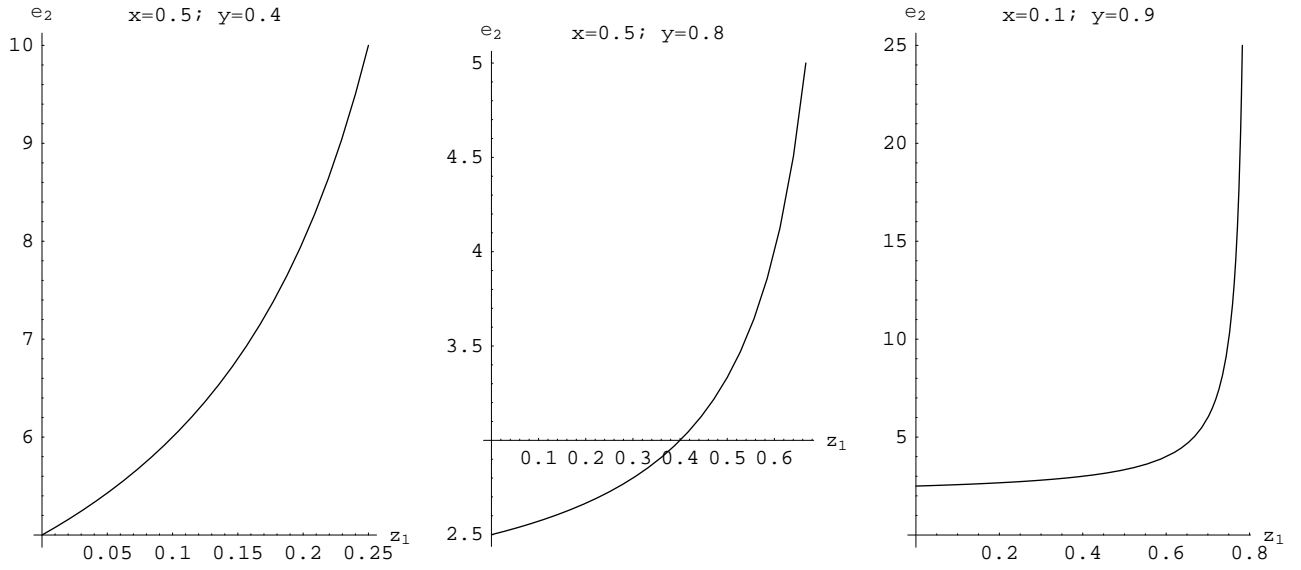
$$e = \frac{2(1-\hat{y})}{1 + (1-\hat{y})^2} \ .$$

Figure 2: The quantity e_2 at different values of x and y as the function of $z_1 = 1 - z$.

Because the quantities e_1 and e_2 depend strongly on z, the determination of the proton structure functions g_1 and g_2 is possible by measurement of z–dependence of the cross–section (15) in a single run without lowering the electron beam energy. The quantity e_1 is proportional to τ and, therefore, is very small at the $HERA$ conditions. Thus, the separation of the g_1 and g_2 in the DIS process with the longitudinally polarized proton is possible in experiments with the target at rest and low values of V (up to 20 GeV^2). At $HERA$ the cross–section of this process can be used to measure the structure function g_1 only. This can be seen from Fig. 1. Contrary, the Fig. 2 shows that the experiments with the tagged photon and perpendicular polarization of the proton can be used to measure both g_1 and g_2 in the wide interval of the energies (provided quantity Q^2 is not large).

3   Radiative corrections

We will restrict ourselves to the model–independent QED radiative corrections related to the radiation of the real and virtual photons by leptons. The remaining sources of RC in the same order of the perturbation theory, such as the virtual corrections with double photon exchange mechanism and bremsstrahlung off the proton and partons, are more involved and model dependent. They are not considered here. Our approach to the calculation of RC is based on the account of all essential Feyman diagrams that describe the observed cross–section in framework of the used approximation. To get rid of cumbersome expressions we will retain in RC the terms that accompanied at least by one power of large logarithms. In the considered case three different types of such logarithms appear

$$L_0 , \quad L_Q = \ln \frac{Q^2}{m^2} , \quad L_\theta = \ln \frac{\theta_0^2}{4} . \tag{18}$$

Besides, in chosen approximation we neglect the terms of the order of θ_0^2, $m^2/\varepsilon^2\theta_0^2$ and m^2/Q^2 in the cross–section.

The total RC to the cross–section (15) includes the contributions due to the virtual and soft photon emission as well as hard photon radiation. We begin with the calculation of the virtual and soft corrections.

3.1   Virtual and soft corrections

To calculate the contribution from the virtual– and soft–photon emission corrections, we start from the expression for the one–loop corrected Compton tensor with a heavy photon for longitudinally polarized electron [13]. For considered here hard collinear initial–state radiation this Compton tensor can be written as

$$L_{\mu\nu}^V = \frac{\alpha}{2\pi}\rho L_{\mu\nu}^B + \frac{\alpha^2}{4\pi^3} \int_\Omega i\varepsilon_{\mu\nu\lambda\rho}q_\lambda k_{1\rho}\frac{d^3k}{\omega}\Big[\frac{T}{-t} + \frac{4m^2(1 - z + z^2)}{t^2}L_Q \ln z\Big] , \tag{19}$$

$$T = \frac{1 + z^2}{1 - z}[2\ln z(l_t - \ln(1 - z) - L_Q) - 2F(z)] + \frac{1 + 2z - z^2}{2(1 - z)} , \quad F(z) = \int_1^{1/z} \frac{dx}{x}\ln|1 - x| ,$$

$$l_t = \ln\frac{-t}{m^2} , \quad \rho = 4(L_Q - 1)\ln\frac{\delta}{m} - L_Q^2 + 3L_Q + 3\ln z + \frac{\pi^2}{3} - \frac{9}{2} ,$$

where δ is the fictitious photon mass, and tensor $L_{\mu\nu}^B$ is defined by Eq. (9).

To eliminate the photon mass we have to add the contribution due to additional soft–photon emission with the energy less than $\Delta\varepsilon$, $\Delta \ll 1$. This contribution has been found in Ref. [14] and the corresponding procedure of the photon–mass elimination has been described in Ref. [15]. The result reads

$$L_{\mu\nu}^{V+S} = L_{\mu\nu}^V(\rho \to \tilde{\rho}) , \tag{20}$$

$$\tilde{\rho} = 2(L_Q - 1)\ln\frac{\Delta^2}{Y} + 3L_Q + 3\ln z - \ln^2 Y - \frac{\pi^2}{3} - \frac{9}{2} + 2Li_2(\cos^2\frac{\theta}{2}) , \quad Y = \frac{\varepsilon_2}{\varepsilon} ,$$

where ε_2 is the scattered–electron energy and θ is the electron scattering angle ($\theta = \widehat{\mathbf{k_1 k_2}}$).

The angular integration with respect to the hard tagged photon over the solid angle of PD gives (in the framework of used accuracy)

$$\mathrm{L}_{\mu\nu}^{V+S} = \left(\frac{\alpha}{2\pi}\right)^2 [\tilde\rho P(z, L_0) + G] dz i \varepsilon_{\mu\nu\lambda\rho} q_\lambda k_{1\rho} \ , \tag{21}$$

$$G = \left\{ \frac{1+z^2}{1-z} [\ln z(L_0 - 2L_Q) - 2F(z)] + \frac{1+2z-z^2}{2(1-z)} \right\} L_0 + \frac{4(1-z+z^2)}{1-z} L_Q \ln z \ .$$

Using the right side of Eq. (21) insdead of $L_{\mu\nu}^{B}$ on the right side of Eq. (6), we derive the contribution of the virtual and soft corrections to the Born cross–section (15) in the following form

$$\frac{d\sigma_{\parallel,\perp}^{V+S}}{\hat{y} d\hat{y} d\hat{x} dz} = \left(\frac{\alpha}{2\pi}\right)^2 [\tilde\rho P(z, L_0) + G] \Sigma_{\parallel,\perp}(\hat{x}, \hat{y}, \widehat{Q}^2) \ , \tag{22}$$

where $\Sigma_{\parallel,\perp}(\hat{x}, \hat{y}, \widehat{Q}^2)$ are defined by Eqs. (16), (17).

3.2 Double hard bremsstrahlung

Let us consider the emission of an additional hard photon with 4–momentum $\tilde{k}$ and the energy more than $\Delta\varepsilon$. To calculate the contribution from the real hard bremsstrahlung, which in our case corresponds to double hard photon emission, with at least one photon seen in the forward PD, we specify three specific kinematical domains:

i) both hard photons hit the forward PD, i.e. both are emitted within a narrow cone around the electron beam ($\widehat{\mathbf{k k_1}}, \widehat{\tilde{\mathbf{k}} \mathbf{k_1}} \leq \theta_0$);

ii) one hard photon is tagged by PD, while the other one is collinear to the outgoing electron momentum ($\widehat{\tilde{\mathbf{k}} \mathbf{k_2}} \leq \theta_0'$, $\theta_0' \ll 1$);

iii) the additional photon is emitted at large angles (i.e. outside the both defined narrow cones) with respect to both incoming and outgoing electron momenta.

The contributions of the regions i) and ii) contain terms quadratic in the large logarithms L_0, L_Q, whereas region iii) contains terms of the order of $L_0 L_\theta$, which can give numerically even larger contribution if $2\theta_0 > \varepsilon\theta_0/m$.

We denote the third kinematic domain as a semi–collinear one. Beyond the leading logarithmic accuracy, the calculation may be performed using the results of paper [16] for leptonic current tensor with longitudinally polarized electron for collinear as well as semi–collinear regions.

The contribution from the kinematic region i), when both hard photons hit PD and every one has the energy more than $\Delta\varepsilon$, can be written as follows

$$\frac{d\sigma_{\parallel,\perp}^{i)}}{\hat{y} d\hat{y} d\hat{x} dz} = \left(\frac{\alpha}{2\pi}\right)^2 L_0 \left\{ [\frac{1}{2} P_\theta^{(2)}(z) + \frac{1+z^2}{1-z}(\ln z - \frac{3}{2} - 2\ln\Delta)] L_0 + \right. \tag{23}$$

$$\left. 7(1-z) - 2(1-z)\ln z + \frac{3+z^2}{2(1-z)}\ln^2 z - 2\frac{3-2z+3z^2}{1-z}\ln\frac{1-z}{\Delta} \right\} \Sigma_{\parallel,\perp}(\hat{x}, \hat{y}, \widehat{Q}^2) \ .$$

The double–logarithmic terms on the right side of Eq. (23) are the same for the polarized and unpolarized cases, whereas one–logarithmic terms are different. In Eq. (23) we use the notation $P_\theta^{(2)}(z)$ for the Θ–part of the second–order electron structure function $D(z, L)$ [17]

$$D(z, L) = \delta(1 - z) + \frac{\alpha}{2\pi}P^{(1)}(z)L + \frac{1}{2}\left(\frac{\alpha}{2\pi}\right)^2 P^{(2)}(z)L^2 + \dots \,,$$

$$P^{(i)}(z) = P_\theta^{(i)}(z)\Theta(1 - z - \Delta) + \delta(1 - z)P_\delta^{(i)} \,, \quad \Delta \to 0 \,,$$

$$P_\theta^{(1)}(z) = \frac{1 + z^2}{1 - z} \,, \quad P_\delta^{(1)} = \frac{3}{2} + 2\ln\Delta \,,$$

$$P_\theta^{(2)}(z) = 2\left[\frac{1 + z^2}{1 - z}\left(2\ln(1 - z) - \ln z + \frac{3}{2}\right) + \frac{1}{2}(1 + z)\ln z - 1 + z\right] \,. \tag{24}$$

To calculate the contribution of the kinematical region $ii)$ we can use the quasi–real electron method to describe the radiation of both collinear photons. This contribution to the observed cross–section depends on the event selection, in other words, on the method of measurement of the scattered electron.

For exclusive event selection, when only the scattered electron is detected, while the photon, that is emitted almost collineary (i.e. within the opening angle $2\theta_0'$ around the momentum of the scattered electron), goes unnoticed or is not taken into account in the determination of the kinematical variables, we have in accordance with Ref. [9]

$$\frac{d\sigma_{\parallel,\perp}^{ii),excl}}{\hat{y}d\hat{y}d\hat{x}dz} = \frac{\alpha^2}{4\pi^2}P(z, L_0)\int\limits_{\Delta/Y}^{y_{1max}}\frac{dy_1}{1 + y_1}\left[\frac{1 + (1 + y_1)^2}{y_1}(\tilde{L} - 1) + y_1\right]\Sigma_{\parallel,\perp}(x_s, y_s, Q_s^2) \,, \tag{25}$$

where y_1 is the energy fraction of the photon, radiated along 3-momentum $\mathbf{k_2}$, relative to the scattered–electron energy ($y_1 = \tilde{\omega}/\varepsilon_2$) and

$$\tilde{L} = \ln\frac{\varepsilon^2\theta_0'^2}{m^2} + 2\ln Y \,, \quad x_s = \frac{xyz(1 + y_1)}{z - (1 - y)(1 + y_1)} \,,$$

$$y_s = \frac{z - (1 - y)(1 + y_1)}{z} \,, \quad Q_s^2 = Q^2z(1 + y_1) \,.$$

The upper limit of the integration in Eq. (25) y_{1max} can be defined from the condition of the inelastic–process availability $p_x^2 = (M + \mu)^2$, where μ is the pion mass. Taking into account that for kinematics $ii)$ $q = zk_1 - (1 + y_1)k_2$ we obtain

$$y_{1max} = \frac{2z\varepsilon[M - \varepsilon_2(1 - c)] - 2M\varepsilon_2 - \mu^2 - 2M\mu}{2\varepsilon_2[M + z\varepsilon(1 - c)]}$$

for the proton target at rest and

$$y_{1max} = \frac{2z - Y(1 + c)}{Y(1 + c)}$$

for the $HERA$ collider, where $c = \cos\theta$. When writing this limit for $HERA$ we neglect the electron energy and the proton mass as compared with the proton beam energy. Note that

parameter θ'_0 for the exclusive event selection is pure auxiliary and escapes the final result when the contribution of the region $iii)$ will be added.

From the experimental point of view more realistic is the calorimeter event selection, when the photon and the electron cannot be distinguished inside narrow cone with the opening angle $2\theta'_0$ along the outgoing–electron momentum direction. Therefore, only the sum of the photon and electron energies can be measured if the photon belongs to this cone. In this case we obtain

$$\frac{d\sigma_{\parallel,\perp}^{ii),cal}}{\hat{y}d\hat{y}d\hat{x}dz} = \frac{\alpha^2}{4\pi^2}P(z,L_0)\int\limits_{\Delta/Y}^{\infty}\frac{dy_1}{(1+y_1)^3}\Big[\frac{1+(1+y_1)^2}{y_1}(\tilde{L}-1)+y_1\Big]\Sigma_{\parallel,\perp}(\hat{x},\hat{y},\widehat{Q}^2) =$$

$$\frac{\alpha^2}{4\pi^2}P(z,L_0)\Big[(\tilde{L}-1)\Big(2\ln\frac{Y}{\Delta}-\frac{3}{2}\Big)+\frac{1}{2}\Big]\Sigma_{\parallel,\perp}(\hat{x},\hat{y},\widehat{Q}^2) \ . \tag{26}$$

For the calorimeter event selection parameter θ'_0 is the physical one, and the final result depends on it (see below).

To calculate the contribution of the region $iii)$ we can use the quasi–real electron method [9] and write the leptonic tensor in this region (that describes the radiation of collinear photon with the energy fraction $1-z$ and noncollinear photon with 4–momentum $\tilde{k}$) in the following form

$$L_{\mu\nu}(k_1,k_2,(1-z)k_1,\tilde{k}) = \frac{\alpha}{2\pi}P(z,L_0)\frac{dz}{z}L_{\mu\nu}(zk_1,k_2,\tilde{k}) \ , \tag{27}$$

$$L_{\mu\nu}(zk_1,k_2,\tilde{k}) = \frac{\alpha}{4\pi^2}\frac{d^3\tilde{k}}{\tilde{\omega}}L_{\mu\nu}^{\gamma}(zk_1,k_2,\tilde{k}) \ , \ L_{\mu\nu}^{\gamma}(zk_1,k_2,\tilde{k}) = 2i\varepsilon_{\mu\nu\lambda\rho}\tilde{q}_{\lambda}\Big[\frac{(\tilde{u}+\tilde{t})z}{\tilde{s}\tilde{t}}k_{1\rho}+\frac{\tilde{s}+\tilde{u}}{\tilde{s}\tilde{t}}k_{2\rho}\Big] \ ,$$

$$\tilde{q} = zk_1-k_2-\tilde{k} \ , \ \tilde{u} = -2zk_2k_1 \ , \ \tilde{s} = 2\tilde{k}k_2 \ , \ \tilde{t} = -2z\tilde{k}k_1 \ .$$

The contraction of the leptonic tensor $L_{\mu\nu}^{\gamma}(k_1,k_2,k)$ and the hadronic one, in the general case of noncollinear photon radiation with 4-momentum k, reads

$$L_{\mu\nu}^{\gamma}(k_1,k_2,k)H_{\mu\nu}^{\parallel} = -\frac{1}{st}\{(2\tau A_t+q^2B)g_1+2\tau[A_t-x'(u+t)B]g_2\}\frac{x'}{q^2} \ , \tag{28}$$

$$L_{\mu\nu}^{\gamma}(k_1,k_2,k)H_{\mu\nu}^{\perp} = -\frac{1}{st}\Big\{\Big[A_s-\frac{uq^2}{V}B-A_t(1-y+\frac{2u\tau}{V})\Big]g_1+\Big[A_s+x'(s+u)B+ \tag{29}$$

$$+(1-y+\frac{2u\tau}{V})(x'(u+t)B-A_t)\Big]g_2\Big\}\frac{x'}{q^2}\sqrt{\frac{M^2}{Q^2}}(1-y+\frac{u\tau}{V})^{-1} \ ,$$

$$A_t = (u+t)^3+(uq^2-st)(u+s) \ , \quad B = (u+t)(2V+\frac{u+t}{x'})+(u+s)(2V(1-y)-\frac{u+s}{x'}) \ ,$$

$$A_s = (u+s)^3+(uq^2-st)(u+t) \ , \quad q = k_1-k_2-k \ , \quad x' = \frac{-q^2}{2p_1q} \ , \quad g_{1,2} = g_{1,2}(x',q^2) \ .$$

The contraction of the *shifted* leptonic tensor $L_{\mu\nu}^{\gamma}(zk_1,k_2,\tilde{k})$, that enters in the definition of the leptonic tensor in the region $iii)$, and hadronic one can be obtained from Eqs. (28) and (29) by the substitution

$$(k_1\ ,k)\to(zk_1\ ,\tilde{k}) \ , \quad (s\ ,t\ ,u\ ,q\ ,x')\to(\tilde{s}\ ,\tilde{t}\ ,\tilde{u}\ ,\tilde{q}\ ,\tilde{x}) \ , \quad \tilde{x} = \frac{-\tilde{q}^2}{2p_1\tilde{q}} \ . \tag{30}$$

We use the approach developed in Ref. [8] to extract the main (proportional to $\ln\theta_0$ and $\ln\theta_0'$) contributions in corresponding cross–section as well as to separate the infrared singularities and write it in the following form

$$\frac{d\sigma^{iii)}_{\parallel,\perp}}{\hat{y}d\hat{x}d\hat{y}dz} = \frac{\alpha^2}{4\pi^2}\Big\{P(z,L_0)\Big[\int\limits_{\Delta}^{x_{1max}}\frac{dx_1[z^2+(z-x_1)^2]}{x_1z(z-x_1)}\ln\frac{2(1-c)}{\theta_0^2}\Sigma_{\parallel,\perp}(x_t,y_t,Q_t^2)+ \tag{31}$$

$$\int\limits_{\Delta/Y}^{y_{1max}}\frac{dy_1[1+(1+y_1)^2]}{y_1(1+y_1)}\ln\frac{2(1-c)}{\theta_0'^2}\Sigma_{\parallel,\perp}(x_s,y_s,Q_s^2)\Big] + \frac{1+z^2}{1-z}L_0Z_{\parallel,\perp}\Big\} ,$$

where

$$x_t = \frac{xy(z-x_1)}{z-x_1+y-1} , \quad y_t = \frac{z-x_1+y-1}{z-x_1} , \quad Q_t^2 = Q^2(z-x_1) .$$

For the proton target at rest

$$x_{1max} = \frac{2z\varepsilon[M-\varepsilon_2(1-c)]-2M\varepsilon_2-\mu^2-2\mu M}{2\varepsilon[M-\varepsilon_2(1-c)]}$$

and for the $HERA$ collider conditions

$$x_{1max} = z - \frac{Y(1+c)}{2} .$$

The dependence on the infrared auxiliary parameter Δ as well as on the angles θ_0 and θ_0' is contained in the first two terms on the right side of Eq. (31), whereas the quantities $Z_{\parallel,\perp}$ do not contain the infrared and collinear singularities. They can be written as

$$Z_{\parallel,\perp} = -\frac{2(1-c)}{zQ^2}\int\limits_0^\infty\frac{du}{1+u^2}\Big\{\Big[\int\limits_0^1\frac{dt_1}{t_1|t_1-a|}\Big[\int\limits_0^{x_m}\frac{dx_1}{x_1}\Phi_{\parallel,\perp}(t_1,t_2(t_1,u)) - \int\limits_0^{Yy_{1m}}\frac{dx_1}{x_1}\Phi_{\parallel,\perp}(a,0)\Big]+ \tag{32}$$

$$\int\limits_0^a\frac{dt_1}{t_1a}\Big[\int\limits_0^{Yy_{1m}}\frac{dx_1}{x_1}\Phi_{\parallel,\perp}(a,0) - \int\limits_0^{x_{1m}}\frac{dx_1}{x_1}\Phi_{\parallel,\perp}(0,a)\Big]\Big\} ,$$

where we use the same notation as in Ref. [8], namely

$$t_{2,1} = \frac{1-c_{1,2}}{2} , \quad a = \frac{1-c}{2} , \quad t_2(t_1,u) = \frac{(a-t_1)^2(1+u^2)}{x_++u^2x_-} , \quad c_{1,2} = \cos\theta_{1,2} , \theta_{1,2} = \widehat{\mathbf{k}\mathbf{k}_{1,2}} ,$$

$$x_\pm = t_1(1-2a)+a\pm 2\sqrt{a(1-a)t_1(1-t_1)} .$$

Quantity $\Phi_{\parallel,\perp}(t_1,t_2)$ reads

$$\Phi_{\parallel,\perp}(t_1,t_2) = \frac{\alpha^2(\tilde{q}^2)\tilde{x}}{\widetilde{Q}^6}G_{\parallel,\perp} , \tag{33}$$

$$G_\parallel = g_1(2\hat{\tau}\widetilde{A}_t+\tilde{q}^2\widetilde{B})+2g_2\hat{\tau}(\widetilde{A}_t-\tilde{x}(\tilde{u}+\tilde{t})\widetilde{B}) , \quad g_{1,2} = g_{1,2}(\tilde{x},\tilde{q}^2) ,$$

$$G_\perp = \sqrt{\frac{M^2}{\widehat{Q}^2}}(1-\hat{y}-\hat{x}\hat{y}\hat{\tau})^{-1}\Big\{g_1\Big[\widetilde{A}_s-\frac{u\tilde{q}^2}{V}\widetilde{B}-\widetilde{A}_t(1-\hat{y}+\frac{2u\hat{\tau}}{V})\Big]+$$

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$$g_2 \Big[\widetilde{A}_s + \tilde{x}(\tilde{s} + \tilde{u})\widetilde{B} + (1 - \hat{y} + \frac{2u\hat{\tau}}{V})[\tilde{x}(\tilde{u} + \tilde{t})\widetilde{B} - \widetilde{A}_t]\Big] \Big\} \ .$$

The upper limit of the integration x_m on the right side of Eq. (32) is

$$\frac{2Mz\varepsilon - 2M\varepsilon_2 - 2z\varepsilon\varepsilon_2(1 - c) - \mu^2 - 2M\mu}{2\varepsilon[M + z\varepsilon(1 - c_1) - \varepsilon_2(1 - c_2)]} \ ; \quad \frac{2z - Y(1 + c)}{1 + c_1}$$

for the proton target at rest and the $HERA$ collider, respectively.

4 Total radiative correction

The total RC to the Born cross–section (15) is defined by the sum of the virtual and soft corrections and hard–photon emission contribution. The last one is different for the exclusive and calorimeter event selection. In the considered approximation it is convenient to write this RC in the following form

$$\frac{d\sigma^{RC}_{\|,\perp}}{\hat{y}d\hat{x}d\hat{y}dz} = \frac{\alpha^2}{4\pi^2}(\Sigma_{i\|,\perp} + \Sigma_{f\|,\perp}) \ . \tag{34}$$

The first term Σ_i is independent on the experimental selection rules for the scattered electron and reads

$$\Sigma_{i\|,\perp} = L_0\Big\{ \frac{1}{2}L_0 P^{(2)}_\theta(z) + \frac{1 + z^2}{1 - z}\Big[5\ln z - 2F(z) + \ln^2 Y - 2\ln z \ln Y - \frac{\pi^2}{3} + 2Li_2\Big(\frac{1 + c}{2}\Big)\Big] + \tag{35}$$

$$\frac{3 + z^2}{2(1 - z)}\ln^2 z - \frac{2(3 - 2z + 3z^2)}{1 - z}\ln(1 - z) + \frac{3 - 20z + z^2}{2(1 - z)}\Big\}\Sigma_{\|,\perp}(\hat{x}, \hat{y}, \widehat{Q}^2) +$$

$$P(z, L_0)\ln\frac{2(1 - c)}{\theta^2_0}\int\limits_0^{u_0}\frac{du}{1 - u}P^{(1)}(1 - u)\Sigma_{\|,\perp}(x_t, y_t, Q^2_t) + \frac{1 + z^2}{1 - z}L_0 Z_{\|,\perp} \ , \quad u_0 = \frac{x_{1max}}{z} \ ,$$

where the quantity $P^{(1)}(x)$ is defined by relations (24) and quantities x_t, y_t, Q^2_t depend on $u = x_1/z$.

The second term on the right side of Eq. (34), denoted by Σ_f, however, explicitly depends on the rule for the event selection. It includes the main effect of the scattered–electron radiation. In the case of exclusive event selection, when only the scattered bare electron is measured, and any photon, collinear respect to its momentum direction, is ignored, this contribution is

$$\Sigma^{excl}_{f\|,\perp} = P(z, L_0)\int\limits_0^{y_{1max}}dy_1[(L_Q + \ln Y - 1)P^{(1)}(\frac{1}{1 + y_1}) + \frac{y_1}{1 + y_1}]\Sigma_{\|,\perp}(x_s, y_s, Q^2_s) \ . \tag{36}$$

As it was mentioned above, in this case the parameter θ'_0, that separate kinematic regions $ii)$ and $iii)$, is not physical, and we see that the final result does not contain it. But the mass singularity, that is connected with the scattered–electron radiation, exhibits itself through L_Q on the right side of Eq. (36).

The situation is quite different for the calorimeter event selection, when the detector cannot distinguish between the events with a bare electron and events where the scattered electron

is accompanied by a hard photon emitted within a narrow cone with the opening angle $2\theta_0'$ around the scattered–electron momentum direction. For such experimental set–up we derive

$$\Sigma_{f\|,\perp}^{cal} = P(z, L_0)\Big[\ln\frac{2(1-c)}{\theta_0'^2}\int\limits_0^{y_{1max}}dy_1 P^{(1)}(\frac{1}{1+y_1})\Sigma_{\|,\perp}(x_s, y_s, Q_s^2) + \frac{1}{2}\Sigma_{\|,\perp}(\hat{x}, \hat{y}, \widehat{Q}^2)\Big] . \quad (37)$$

For the calorimeter set–up the parameter θ_0' defines the rule of the event selection and has, therefore, the physical sense. The final result depends on it. However, the mass singularity due to photon emission by the final electron is cancelled in accordance with the Kinoshita–Lee–Nauenberg theorem [18]. The absence of the mass singularity indicates clearly that term, containing $\ln\theta_0'$ on the right side of Eq. (37), arises due to contribution of the kinematical region iii) where the scattered electron and the photon, radiated from the final–state, are well separated. That is why no question appears to determine quantity ε_2 that enters in the expression for y_{1max}.

The comparison of our analytical results for RC due to the real–and virtual–photon emission with the analogous calculations for unpolarized case [8] shows that, within the leading-log accuracy (double–logarithmic terms in our case), this RC are the same for the spin–dependent and spin–independent parts of the cross–section of the radiative DIS process (1). The difference appears on the level of the next–to–leading–log accuracy (single–logarithmic terms in our case). That is true for the photonic corrections in arbitrary order of the perturbation theory.

Note that the correction to the usually measured asymmetry, which is the ratio of the spin–dependent part of the cross–section to the spin–independent one, is not large because the main factorized contribution due to the virtual– and soft–photon emission trends to cancellation in this case. If experimental information about the spin observables is extracted directly from the spin–dependent part of the cross–section (for corresponding experimental method see Ref. [19]) such cancellation does not take place and factorized correction gives the basic contribution.

5   The case of quasi–elastic scattering

In previous Sections we considered the tagged–photon events in the DIS process. Such events can be used to measure the spin–dependent proton structure functions g_1 and g_2 in a single run without lowering the electron beam energy. In the quasi–elastic limiting case, when the target proton is scattered elastically

$$e^-(k_1) + p(p_1) \rightarrow e^-(k_2) + \gamma(k) + p(p_2) , \quad (38)$$

the tagged–photon events can be used also to measure the proton electromagnetic form factors G_E and G_M. Our final results, obtained in Section 4, can be applied by using connection between the spin–dependent proton structure functions g_1 and g_2 and the proton electromagnetic form factors in this limit as given by relations (11). Therefore, in this case we can use all formulae of Section 4 with substitution $\Sigma_{\|,\perp}^{el}$ and $G_{\|,\perp}^{el}$ instead of $\Sigma_{\|,\perp}$ and $G_{\|,\perp}$ (that enters in definition of $Z_{\|,\perp}$), respectively

$$\Sigma_\|^{el}(x, y, Q^2) = \frac{4\pi\alpha^2(Q^2)}{y(4M^2+Q^2)}[4\tau(\tau+1-\frac{1}{y})G_M G_E - (1-\frac{y}{2})(1+2\tau)G_M^2]\delta(1-x) , \quad (39)$$

$$\Sigma_\perp^{el}(x,y,Q^2) = \frac{8\pi\alpha^2(Q^2)}{y(4M^2+Q^2)}\sqrt{\frac{M^2}{Q^2}[1-y(1+\tau)]}[(1-\frac{y}{2})G_M^2 - (1+2\tau)G_M G_E]\delta(1-x) \ , \ (40)$$

$$G_{\parallel,\perp}^{el} = \frac{\widetilde{Q}^2}{4M^2+\widetilde{Q}^2}[D_{\parallel,\perp}G_M^2 + E_{\parallel,\perp}G_M G_E]\delta(1-\tilde{x}) \ , \qquad (41)$$

$$D_\parallel = \bar{B}[\tilde{q}^2 + 2\hat{\tau}(\tilde{u}+\tilde{t})] \ , \ E_\parallel = 2\hat{\tau}[(1+\frac{4M^2}{\widetilde{Q}^2})\widetilde{A}_t - \bar{B}(2\hat{V}+\tilde{u}+\tilde{t})] \ ,$$

$$D_\perp = -K\bar{B}\Big[\frac{u\tilde{q}^2}{V} + \tilde{s} + \tilde{u} + (\tilde{u}+\tilde{t})(1-\hat{y}+\frac{2u\hat{\tau}}{V})\Big] \ ,$$

$$E_\perp = K\{(1+\frac{4M^2}{\widetilde{Q}^2})[\widetilde{A}_s - (1-\hat{y}+\frac{2u\hat{\tau}}{V})\widetilde{A}_t] + \bar{B}[\tilde{s}+\tilde{u}(1+4\hat{\tau})+(\tilde{u}+\tilde{t})(1-\hat{y}+\frac{2u\hat{\tau}}{V})]\} \ ,$$

where

$$\bar{B} = (\tilde{u}+\tilde{t})(2\hat{V}+\tilde{u}+\tilde{t}) + (\tilde{u}+\tilde{s})(2\hat{V}(1-\hat{y})-\tilde{u}-\tilde{s}) \ , \ K = \sqrt{\frac{M^2}{\widehat{Q}^2}(1-\hat{y}-\hat{x}\hat{y}\hat{\tau})^{-1}}$$

and form factors on the right side of Eq. (41) depend on $\tilde{q}^2$.

The description of the form factors is very important test for every theoretical model of the strong interaction [20]. The magnetic form factor of the proton G_M is known with a good accuracy in the wide interval of the momentum transfer, while the data about the electric one G_E are very poor. The recent experiment in the Jefferson Lab on the measurement of the ratio of the recoil–proton polarizations, performed by the Hall A Collaboration [21], improves the situation in the region up to $Q^2 \simeq 3.5 GeV^2$, but the region of the higher momentum transfer remains unstudied. The use of the radiative events (38), with both polarized and unpolarized proton target, on accelerators with the high–intensity electron beam (for example, $CEBAF$) can open the new possibilities in the measurement of G_E as compared with both the Rosenbluth method [22] and method based on the measurement of the recoil–proton polarizations ratio [23].

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